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CALCULATION OF THE CHANGE IN IMPEDANCE OF A COIL LOCATED ABOVE A CONDUCTING MEDIUM WITH A FLAW

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ABSTRACT

Semi-analytical solution of one eddy current testing problem is presented in the paper. A coil with alternating current is located above a conducting half-space with a flaw in the form of a circular cylinder. The axis of the coil coincides with the axis of the cylinder. The problem is solved by the method of truncated eigenfunction expansions. The change in impedance of the coil is computed for different values of the parameters of the problem.

Key words: eddy currents, Maxwell's equations, TREE method

1. Introduction

Solutions of eddy current testing problems for the case where a multilayer conducting medium is unbounded in one or two spatial dimensions are well-known in the literature [1]-[3]. Methods of integral transforms (for example, Hankel or Fourier transforms) are used in such cases in order to solve the system of the Maxwell's equations analytically. The change in impedance of the coil is represented by means of an improper integral containing special functions. Flaws in the conducting medium are usually of finite size so that analytical solution of the corresponding problem with flaws cannot be found. One example of such a case is spot welding [4].
If a conducting medium contains a finite region whose properties differ from the properties of the surrounding medium then the corresponding problem can be solved either by numerical methods [5] or using perturbation expansions. An example of using an approximate method (the layer approximation) for the solution of eddy current problems with flaws is given in [6]. One of the shortcomings of perturbation methods is the inability to control errors associated with perturbation expansions.

A quasi-analytical approach for the solution of eddy current testing problems for finite domains (the TREE method) is suggested recently in [3]. It is assumed in the TREE method that the vector potential is zero at a sufficiently large radial distance $r = b$ from an eddy current coil. By choosing the value of b the user can control errors in using the TREE method. Different forms of the boundary conditions and suggestions for the selection of the value of b are given in [3].

In the present paper we consider a coil with alternating current located above a conducting half-space with a flaw in the form of a circular cylinder. Such a model can be used to test the quality of spot welding [4], [7], [8] by eddy current methods. The axis of the coil coincides with the axis of the flaw. We use the TREE method to solve the problem. Calculations of the change in impedance are performed with Mathematica for different values of the parameters of the problem.

2. Mathematical formulation of the problem

Consider a cylindrical coil with alternating current located above a conducting half-space with a flaw in the form of a cylinder of finite dimensions (see Fig. 1). The inner and outer radii of the coil are r_1 and r_2, respectively. The bottom of the coil is located at the distance z_1 from the conducting half-space. The height of the coil is $z_2 - z_1$ and the number of turns is N.

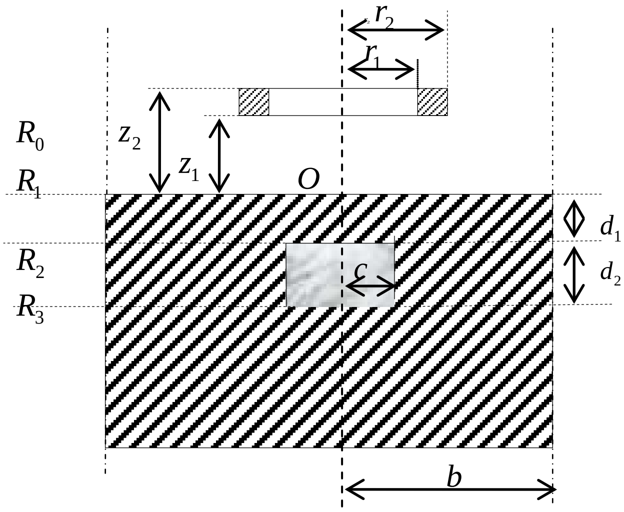

Fig. 1. A coil of finite dimensions above a conducting half-space with a flaw.

In order to obtain the change in impedance of the coil we need to solve the problem for a single-turn coil and then use the superposition principle. The amplitude of the vector potential (due to azimuthal symmetry) has only one nonzero component in the azimuthal direction in each of the regions $R_0 - R_3$ (the nonzero components of the vector potential in regions R_0, R_1, R_2 and R_3 are denoted by A_0, A_1, A_2 and A_3, respectively). Assuming that a single-turn coil of radius r_0 is located at distance h above the conducting half-space we obtain the system of equations for the components of the vector potential in regions $R_i, i = 0,1,2,3$

$$\frac{\partial^2 A_0}{\partial r^2} + \frac{1}{r}\frac{\partial A_0}{\partial r} - \frac{A_0}{r^2} + \frac{\partial^2 A_0}{\partial z^2} = -\mu_0 I \delta(r - r_0)\delta(z - h), \tag{1}$$

$$\frac{\partial^2 A_1}{\partial r^2} + \frac{1}{r}\frac{\partial A_1}{\partial r} - \frac{A_1}{r^2} - j\omega\sigma_1\mu_0 A_1 + \frac{\partial^2 A_1}{\partial z^2} = 0, \tag{2}$$

$$\frac{\partial^2 A_2}{\partial r^2} + \frac{1}{r}\frac{\partial A_2}{\partial r} - \frac{A_2}{r^2} - j\omega\sigma_2\mu_0 A_2 + \frac{\partial^2 A_2}{\partial z^2} = 0, \tag{3}$$

$$\frac{\partial^2 A_3}{\partial r^2} + \frac{1}{r}\frac{\partial A_3}{\partial r} - \frac{A_3}{r^2} - j\omega\sigma_1\mu_0 A_3 + \frac{\partial^2 A_3}{\partial z^2} = 0, \tag{4}$$

where σ_1 is the conductivity of the half-space and $\sigma_2 = \begin{cases} \sigma_1, & c < r < b \\ \sigma, & 0 \le r < c \end{cases}$ is the conductivity of region

R_2 and ω is the frequency.

The boundary conditions are

$$A_i\,|_{r=b}=0, \quad i=0,1,3, \;\; A_2^{con}\,|_{r=b}=0, \tag{5}$$

$$A_0\,|_{z=0}=A_1\,|_{z=0}, \quad \frac{\partial A_0}{\partial z}|_{z=0}=\frac{\partial A_1}{\partial z}|_{z=0}, \quad 0\le r\le b, \tag{6}$$

$$A_1\,|_{z=-d_1}=A_2^{cc}\,|_{z=-d_1}, \quad \frac{\partial A_1}{\partial z}|_{z=-d_1}=\frac{\partial A_2^{cc}}{\partial z}|_{z=-d_1}, \quad 0\le r\le c, \tag{7}$$

$$A_1\,|_{z=-d_1}=A_2^{con}\,|_{z=-d_1}, \quad \frac{\partial A_1}{\partial z}|_{z=-d_1}=\frac{\partial A_2^{con}}{\partial z}|_{z=-d_1}, \quad c\le r\le b, \tag{8}$$

$$A_2^{cc}\,|_{z=-d_1-d_2}=A_3\,|_{z=-d_1-d_2}, \quad \frac{\partial A_2^{cc}}{\partial z}|_{z=-d_1-d_2}=\frac{\partial A_2}{\partial z}|_{z=-d_1-d_2}, \quad 0\le r\le c, \tag{9}$$

$$A_2^{con}\,|_{z=-d_1-d_2}=A_3\,|_{z=-d_1-d_2}, \quad \frac{\partial A_2^{con}}{\partial z}|_{z=-d_1-d_2}=\frac{\partial A_3}{\partial z}|_{z=-d_1-d_2}, \quad c\le r\le b, \tag{10}$$

$$A_2^{con}\,|_{r=c}=A_2^{cc}\,|_{r=c}, \quad \frac{\partial A_2^{con}}{\partial r}|_{r=c}=\frac{\partial A_2^{cc}}{\partial r}|_{r=c}. \tag{11}$$

The abbreviations “cc” and “con” are used in region R_2 to denote the conducting cylinder and homogeneous conducting half-space, respectively. Problem (1)-(11) is solved by means of the TREE method in [9].

3. The change in impedance of the coil

It is shown in [9] that the induced vector potential in air due to the presence of the conducting half-space with the flaw has the form

$$A_0^{ind}(r,z,r_0,h)=\sum_{j=1}^{n} D_{2j}e^{-\lambda_j z}J_1(\lambda_j r), \tag{12}$$

where

$\lambda_i=\alpha_i/b$, α_i are the roots of the equation $J_1(\alpha_i)=0$,

$$D_{2j}=D_{4j}+D_{5j}-\mu_0 I r_0\frac{J_1(\lambda_j r_0)e^{-\lambda_j h}}{\lambda_j b^2 J_0^2(\lambda_j b)}, \tag{13}$$

$$D_{4j}=\sum_{i=1}^{n}\left\{\left(\frac{(p_j+p_{1i})e^{(p_j-p_{1i})d_1}a_{ji}}{p_j b^2 J_0^2(\lambda_j b)}\hat{D}_{6i}+\frac{(p_j-p_{1i})e^{(p_j+p_{1i})d_1}a_{ji}}{p_j b^2 J_0^2(\lambda_j b)}\hat{D}_{8i}\right)\right\}, \tag{14}$$

$$D_{5j}=\sum_{i=1}^{n}\left\{\left(\frac{(p_j-p_{1i})e^{-(p_j+p_{1i})d_1}a_{ji}}{p_j b^2 J_0^2(\lambda_j b)}\hat{D}_{6i}+\frac{(p_j+p_{1i})e^{-(p_j-p_{1i})d_1}a_{ji}}{p_j b^2 J_0^2(\lambda_j b)}\hat{D}_{8i}\right)\right\}, \tag{15}$$

$p_{1i}=\sqrt{\lambda_i^2+j\omega\sigma_1\mu_0}$, p_i are the complex roots of the equation

$$p_i J_1'(p_i c)T_1(q_i c)=q_i T_1'(q_i c)J_1(p_i c), \tag{16}$$

$q_i=\sqrt{p_i^2-j\omega\sigma\mu_0}$ and $\hat{D}_{6i}$ and $\hat{D}_{8i}$ are the solutions of the following system of algebraic equations

$$\sum_{i=1}^{\infty}\left(\begin{array}{l}\left((\lambda_j+p_j)(p_j+p_{1i})e^{(p_j-p_{1i})d_1}+(\lambda_j-p_j)(p_j-p_{1i})e^{-(p_j+p_{1i})d_1}\right)a_{ji}\hat{D}_{6i}+\\ +\left((\lambda_j+p_j)(p_j-p_{1i})e^{(p_j+p_{1i})d_1}+(\lambda_j-p_j)(p_j+p_{1i})e^{-(p_j-p_{1i})d_1}\right)a_{ji}\hat{D}_{8i}\end{array}\right)= \tag{17}$$
$$=2p_j\mu_0 I r_0 J_1(\lambda_j r_0)e^{-\lambda_j h}$$

$$\sum_{i=1}^{\infty}\left((p_j-p_{1i})e^{-p_{1i}d_3}a_{ji}\hat{D}_{6i}+(p_j+p_{1i})e^{p_{1i}d_3}a_{ji}\hat{D}_{8i}\right)=0.$$

The induced vector potential in air due to currents in the whole coil is obtained as follows

$$A_{0coil}^{ind}(r,z) = \int_{r_1}^{r_2}\int_{z_1}^{z_2} A_0^{ind}(r,z,r_0,h)\,dr_0\,dh\,. \tag{18}$$

There is an important difference between the calculations for a single-turn coil and coil of finite dimensions. For the case of a single-turn coil the geometrical parameters of the coil (r_0 and h) are constants and the numerical values of r_0 and h can be used to solve system (17). If a coil has finite dimensions then one needs to integrate the solution with respect to r_0 and h. Thus, numerical values to the parameters r_0 and h in (17) cannot be assigned. In this case, in order to obtain the induced vector potential of a coil of finite dimensions we need to solve (17) in matrix form. Solving the second equation in (17) we obtain

$$\vec{X}_2 = -A_{22}^{-1}A_{21}\vec{X}_1, \tag{19}$$

where the block matrices are defined as follows

$$A_{11} = \begin{pmatrix} \left((\lambda_1+p_1)(p_1+p_{1_1})e^{(p_1-p_{1_1})d_1} + (\lambda_1-p_1)(p_1-p_{1_1})e^{-(p_1+p_{1_1})d_1}\right)a_{11} \ldots\ldots\ldots\ldots \\ \ldots\left((\lambda_2+p_2)(p_2+p_{1_2})e^{(p_2-p_{1_2})d_1} + (\lambda_2-p_2)(p_2-p_{1_2})e^{-(p_2+p_{1_2})d_1}\right)a_{22} \ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\left((\lambda_n+p_n)(p_n+p_{1_n})e^{(p_n-p_{1_n})d_1} + (\lambda_n-p_n)(p_n-p_{1_n})e^{-(p_n+p_{1_n})d_1}\right)a_{nn} \end{pmatrix}$$

$$A_{12} = \begin{pmatrix} \left((\lambda_1+p_1)(p_1-p_{1_1})e^{(p_1+p_{1_1})d_1} + (\lambda_1-p_1)(p_1+p_{1_1})e^{-(p_1-p_{1_1})d_1}\right)a_{11} \ldots\ldots\ldots\ldots \\ \ldots\left((\lambda_2+p_2)(p_2-p_{1_2})e^{(p_2+p_{1_2})d_1} + (\lambda_2-p_2)(p_2+p_{1_2})e^{-(p_2-p_{1_2})d_1}\right)a_{22} \ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\left((\lambda_n+p_n)(p_n-p_{1_n})e^{(p_n+p_{1_n})d_1} + (\lambda_n-p_n)(p_n+p_{1_n})e^{-(p_n-p_{1_n})d_1}\right)a_{nn} \end{pmatrix}$$

$$A_{21} = \begin{pmatrix} (p_1-p_{1_1})e^{-p_{1_1}d_3}a_{11} & (p_1-p_{1_2})e^{-p_{1_2}d_3}a_{12} \ldots\ldots & (p_1-p_{1_n})e^{-p_{1_n}d_3}a_{1n} \\ (p_2-p_{1_1})e^{-p_{1_1}d_3}a_{21} & (p_2-p_{1_2})e^{-p_{1_2}d_3}a_{22} \ldots\ldots & (p_2-p_{1_n})e^{-p_{1_n}d_3}a_{2n} \\ \ldots\ldots\ldots\ldots\ldots\ldots & & \\ (p_n-p_{1_1})e^{-p_{1_1}d_3}a_{n1} & (p_n-p_{1_2})e^{-p_{1_2}d_3}a_{n2} \ldots\ldots & (p_n-p_{1_n})e^{-p_{1_n}d_3}a_{nn} \end{pmatrix}$$

$$A_{22} = \begin{pmatrix} (p_1+p_{1_1})e^{p_{1_1}d_3}a_{11} & (p_1+p_{1_2})e^{p_{1_2}d_3}a_{12} \ldots\ldots & (p_1+p_{1_n})e^{p_{1_n}d_3}a_{1n} \\ (p_2+p_{1_1})e^{p_{1_1}d_3}a_{21} & (p_2+p_{1_2})e^{p_{1_2}d_3}a_{22} \ldots\ldots & (p_2+p_{1_n})e^{p_{1_n}d_3}a_{2n} \\ \ldots\ldots\ldots\ldots\ldots\ldots & & \\ (p_n+p_{1_1})e^{p_{1_1}d_3}a_{n1} & (p_n+p_{1_2})e^{p_{1_2}d_3}a_{n2} \ldots\ldots & (p_n+p_{1_n})e^{p_{1_n}d_3}a_{nn} \end{pmatrix}$$

Substituting (19) into the first equation in (17) gives

$$\left(A_{11} - A_{12}A_{22}^{-1}A_{21}\right)\vec{X}_1 = \vec{b}. \tag{20}$$

It follows from (19) and (20) that

$$\begin{cases} \vec{X}_1 = \left(A_{11} - A_{12}A_{22}^{-1}A_{21}\right)^{-1}\vec{b} \\ \vec{X}_2 = -A_{22}^{-1}A_{21}\left(A_{11} - A_{12}A_{22}^{-1}A_{21}\right)^{-1}\vec{b} \end{cases} \tag{21}$$

Using matrix notations we rewrite equation (13) in matrix form:

$$\vec{D}_2 = Y\vec{b}, \tag{22}$$

where

$$\vec{D}_2 = \begin{pmatrix} D_{2_1} \\ D_{2_2} \\ \dots \\ D_{2_n} \end{pmatrix}, \qquad Y = \left(B_{11} + B_{21} - \left(B_{12} + B_{22}\right)A_{22}^{-1}A_{21}\right)\left(A_{11} - A_{12}A_{22}^{-1}A_{21}\right)^{-1} - C_{diag}, \tag{23}$$

$$B_{11} = \begin{pmatrix} \frac{\left(p_1 + p_{1_1}\right)e^{\left(p_1 - p_{1_1}\right)d_1}a_{11}}{p_1 b^2 J_0^2(\lambda_1 b)} & \frac{\left(p_1 + p_{1_2}\right)e^{\left(p_1 - p_{1_2}\right)d_1}a_{12}}{p_1 b^2 J_0^2(\lambda_1 b)} & \dots & \frac{\left(p_1 + p_{1_n}\right)e^{\left(p_1 - p_{1_n}\right)d_1}a_{1n}}{p_1 b^2 J_0^2(\lambda_1 b)} \\ \frac{\left(p_2 + p_{1_1}\right)e^{\left(p_2 - p_{1_1}\right)d_1}a_{21}}{p_2 b^2 J_0^2(\lambda_2 b)} & \frac{\left(p_2 + p_{1_2}\right)e^{\left(p_2 - p_{1_2}\right)d_1}a_{22}}{p_2 b^2 J_0^2(\lambda_2 b)} & \dots & \frac{\left(p_2 + p_{1_n}\right)e^{\left(p_2 - p_{1_n}\right)d_1}a_{2n}}{p_2 b^2 J_0^2(\lambda_2 b)} \\ & \dots\dots\dots\dots\dots & & \\ \frac{\left(p_n + p_{1_1}\right)e^{\left(p_n - p_{1_1}\right)d_1}a_{n1}}{p_n b^2 J_0^2(\lambda_n b)} & \frac{\left(p_n + p_{1_2}\right)e^{\left(p_n - p_{1_2}\right)d_1}a_{n2}}{p_n b^2 J_0^2(\lambda_n b)} & \dots & \frac{\left(p_n + p_{1_n}\right)e^{\left(p_n - p_{1_n}\right)d_1}a_{nn}}{p_n b^2 J_0^2(\lambda_n b)} \end{pmatrix},$$

$$B_{12} = \begin{pmatrix} \frac{\left(p_1 - p_{1_1}\right)e^{\left(p_1 + p_{1_1}\right)d_1}a_{11}}{p_1 b^2 J_0^2(\lambda_1 b)} & \frac{\left(p_1 - p_{1_2}\right)e^{\left(p_1 + p_{1_2}\right)d_1}a_{12}}{p_1 b^2 J_0^2(\lambda_1 b)} & \dots & \frac{\left(p_1 - p_{1_n}\right)e^{\left(p_1 + p_{1_n}\right)d_1}a_{1n}}{p_1 b^2 J_0^2(\lambda_1 b)} \\ \frac{\left(p_2 - p_{1_1}\right)e^{\left(p_2 + p_{1_1}\right)d_1}a_{21}}{p_2 b^2 J_0^2(\lambda_2 b)} & \frac{\left(p_2 - p_{1_2}\right)e^{\left(p_2 + p_{1_2}\right)d_1}a_{22}}{p_2 b^2 J_0^2(\lambda_2 b)} & \dots & \frac{\left(p_2 - p_{1_n}\right)e^{\left(p_2 + p_{1_n}\right)d_1}a_{2n}}{p_2 b^2 J_0^2(\lambda_2 b)} \\ & \dots\dots\dots\dots\dots & & \\ \frac{\left(p_n - p_{1_1}\right)e^{\left(p_n + p_{1_1}\right)d_1}a_{n1}}{p_n b^2 J_0^2(\lambda_n b)} & \frac{\left(p_n - p_{1_2}\right)e^{\left(p_n + p_{1_2}\right)d_1}a_{n2}}{p_n b^2 J_0^2(\lambda_n b)} & \dots & \frac{\left(p_n - p_{1_n}\right)e^{\left(p_n + p_{1_n}\right)d_1}a_{nn}}{p_n b^2 J_0^2(\lambda_n b)} \end{pmatrix},$$

$$B_{21} = \begin{pmatrix} \frac{\left(p_1 - p_{1_1}\right)e^{-\left(p_1 + p_{1_1}\right)d_1}a_{11}}{p_1 b^2 J_0^2(\lambda_1 b)} & \frac{\left(p_1 - p_{1_2}\right)e^{-\left(p_1 + p_{1_2}\right)d_1}a_{12}}{p_1 b^2 J_0^2(\lambda_1 b)} & \dots & \frac{\left(p_1 - p_{1_n}\right)e^{-\left(p_1 + p_{1_n}\right)d_1}a_{1n}}{p_1 b^2 J_0^2(\lambda_1 b)} \\ \frac{\left(p_2 - p_{1_1}\right)e^{-\left(p_2 + p_{1_1}\right)d_1}a_{21}}{p_2 b^2 J_0^2(\lambda_2 b)} & \frac{\left(p_2 - p_{1_2}\right)e^{-\left(p_2 + p_{1_2}\right)d_1}a_{22}}{p_2 b^2 J_0^2(\lambda_2 b)} & \dots & \frac{\left(p_2 - p_{1_n}\right)e^{-\left(p_2 + p_{1_n}\right)d_1}a_{2n}}{p_2 b^2 J_0^2(\lambda_2 b)} \\ & \dots\dots\dots\dots\dots & & \\ \frac{\left(p_n - p_{1_1}\right)e^{-\left(p_n + p_{1_1}\right)d_1}a_{n1}}{p_n b^2 J_0^2(\lambda_n b)} & \frac{\left(p_n - p_{1_2}\right)e^{-\left(p_n + p_{1_2}\right)d_1}a_{n2}}{p_n b^2 J_0^2(\lambda_n b)} & \dots & \frac{\left(p_n - p_{1_n}\right)e^{-\left(p_n + p_{1_n}\right)d_1}a_{nn}}{p_n b^2 J_0^2(\lambda_n b)} \end{pmatrix},$$

$$B_{22} = \begin{pmatrix} \frac{\left(p_1 + p_{1_1}\right)e^{-\left(p_1 - p_{1_1}\right)d_1}a_{11}}{p_1 b^2 J_0^2(\lambda_1 b)} & \frac{\left(p_1 + p_{1_2}\right)e^{-\left(p_1 - p_{1_2}\right)d_1}a_{12}}{p_1 b^2 J_0^2(\lambda_1 b)} & \dots & \frac{\left(p_1 + p_{1_n}\right)e^{-\left(p_1 - p_{1_n}\right)d_1}a_{1n}}{p_1 b^2 J_0^2(\lambda_1 b)} \\ \frac{\left(p_2 + p_{1_1}\right)e^{-\left(p_2 - p_{1_1}\right)d_1}a_{21}}{p_2 b^2 J_0^2(\lambda_2 b)} & \frac{\left(p_2 + p_{1_2}\right)e^{-\left(p_2 - p_{1_2}\right)d_1}a_{22}}{p_2 b^2 J_0^2(\lambda_2 b)} & \dots & \frac{\left(p_2 + p_{1_n}\right)e^{-\left(p_2 - p_{1_n}\right)d_1}a_{2n}}{p_2 b^2 J_0^2(\lambda_2 b)} \\ & \dots\dots\dots\dots\dots & & \\ \frac{\left(p_n + p_{1_1}\right)e^{-\left(p_n - p_{1_1}\right)d_1}a_{n1}}{p_n b^2 J_0^2(\lambda_n b)} & \frac{\left(p_n + p_{1_2}\right)e^{-\left(p_n - p_{1_2}\right)d_1}a_{n2}}{p_n b^2 J_0^2(\lambda_n b)} & \dots & \frac{\left(p_n + p_{1_n}\right)e^{-\left(p_n - p_{1_n}\right)d_1}a_{nn}}{p_n b^2 J_0^2(\lambda_n b)} \end{pmatrix},$$

$$C_{diag} = \begin{pmatrix} \frac{1}{2p_1\lambda_1 b^2 J_0^2(\lambda_1 b)} \; 0 \; 0 \; 0 \; \dots \; 0 \\ 0 \; \frac{1}{2p_2\lambda_2 b^2 J_0^2(\lambda_2 b)} \; 0 \; 0 \; \dots \; 0 \\ \dots\dots\dots\dots \\ 0 \; 0 \; \dots \; 0 \; 0 \; \frac{1}{2p_n\lambda_n b^2 J_0^2(\lambda_n b)} \end{pmatrix}, \vec{b} = \begin{pmatrix} 2p_1\mu_0 I r_0 J_1(\lambda_1 r_0)e^{-\lambda_1 h} \\ 2p_2\mu_0 I r_0 J_1(\lambda_2 r_0)e^{-\lambda_2 h} \\ \dots\dots\dots \\ 2p_n\mu_0 I r_0 J_1(\lambda_n r_0)e^{-\lambda_n h} \end{pmatrix} \tag{24}$$

Formula (12) for the induced vector potential of a single-turn coil can be rewritten in the form

$$A_0^{ind}(r,z,r_0,h) = \vec{D}_2^T \vec{f}, \quad (25)$$

where

$$\vec{f} = \begin{pmatrix} J_1(\lambda_1 r)e^{-\lambda_1 z} \\ J_1(\lambda_2 r)e^{-\lambda_2 z} \\ \dots \\ J_1(\lambda_n r)e^{-\lambda_n z} \end{pmatrix}.$$

Substituting (25) into (12) we obtain the induced vector potential in air due to the presence of the conducting medium shown in Fig. 1 (the current amplitude I in this case is replaced by the current density $\frac{NI}{(r_2-r_1)(z_2-z_1)}$):

$$A_{0coil}^{ind}(r,z) = \frac{\mu_0 NI}{(r_2-r_1)(z_2-z_1)} \sum_{j=1}^{n} f_j \sum_{i=1}^{n} Y_{ji} \frac{\left(e^{-\lambda_i z_1} - e^{-\lambda_i z_2}\right)}{\lambda_i^3} \int_{\lambda_i r_1}^{\lambda_i r_2} \xi J_1(\xi) d\xi. \quad (26)$$

The integral with respect to ξ in (26) can be computed in terms of the Bessel and Struve functions [10] as follows

$$\int_{\lambda_i r_1}^{\lambda_i r_2} \xi J_1(\xi) d\xi = \left\{ \frac{\pi}{2} \xi \left[J_0(\xi) H_1(\xi) - J_1(\xi) H_0(\xi) \right] \right\} \Bigg|_{\xi=\lambda_i r_1}^{\xi=\lambda_i r_2} \quad (27)$$

The induced change in impedance of a coil of finite dimensions is calculated by means of the following formula [3]:

$$Z^{ind} = \frac{2\pi j\omega}{I} \frac{N}{(r_2-r_1)(z_2-z_1)} \int_{r_1}^{r_2} \int_{z_1}^{z_2} r A_{0coil}^{ind}(r,z) dr dz. \quad (28)$$

Using (26) and (28) we obtain the induced change in impedance of the coil in the form

$$Z^{ind} = \frac{2j\omega\pi\mu_0 N^2}{(r_2-r_1)^2(z_2-z_1)^2} \sum_{j=1}^{n} \frac{\left(e^{-\lambda_j z_1} - e^{-\lambda_j z_2}\right)}{\lambda_j^3} \int_{\lambda_j r_1}^{\lambda_j r_2} \xi J_1(\xi) d\xi \sum_{i=1}^{n} Y_{ji} \frac{\left(e^{-\lambda_i z_1} - e^{-\lambda_i z_2}\right)}{\lambda_i^3} \int_{\lambda_i r_1}^{\lambda_i r_2} \xi J_1(\xi) d\xi \quad (29)$$

Formula (29) is used to compute the change in impedance of the coil. Calculations are performed with “Mathematica”. The following parameters of the problem are selected: $\mu_0 = 4 \cdot 10^{-7}\pi$, $\sigma_1 = 18.5$ Ms/m, $\sigma_2 = 3$ Ms/m, $c = 2.2$ mm, $r_2 = 5.5$ mm, $r_1 = 3.5$ mm , $z_1 = 0.3$ mm, $z_2 = 2.6$ mm, $d_1 = 0.7$ mm, $d_2 = 0.3$ mm, $b = 55$ mm, $N = 200$. The change in impedance is computed for the following seven frequencies: $f = 1000$, 2000, 3000, 4000, 5000, 6000, and 7000 Hz. The results of calculations are shown in Fig. 2.The calculated points (from top to bottom) correspond to the seven frequencies (from smallest to largest). The upper limit of the summation index in (28) is fixed at $n = 62$. Comparison of the computational results obtained for other values of n showed that the chosen value of 62 is quite satisfactory in terms of calculation accuracy. Several computational steps are necessary in order to calculate the induced change in impedance. First, the set of eigenvalues λ_i has to be calculated. This can easily be done in Mathematica using a built-in routine BesselZeros. Second, a set of complex roots of (16) should be computed. Calculations are based on the method described in [11,12]. Third, several systems of linear equations have to be solved in order to determine expansion coefficients. Finally, the change in impedance is computed using (29).

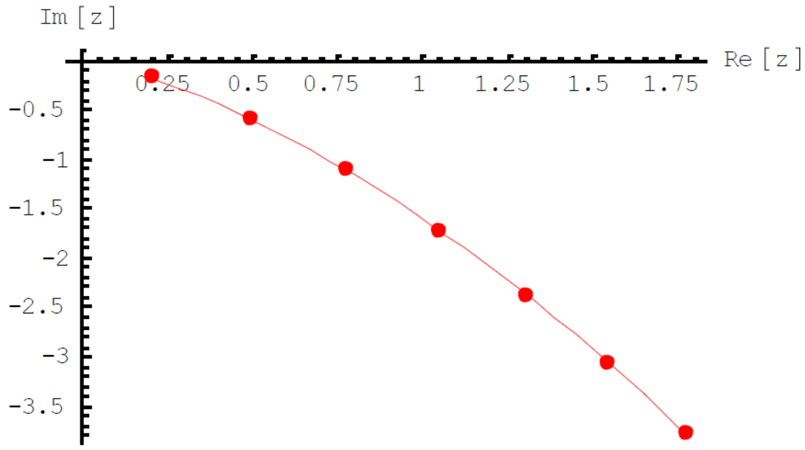

Fig.2. The change in impedance of the coil for seven frequencies.

4. Conclusions

Semi-analytical solution of eddy current problem is presented in the paper. A coil with alternating current is located above a conducting half-space with a flaw in the form of a conducting cylinder whose axis coincides with the axis of the coil. The solution is found by means of the TREE method. Results of numerical computations are presented.

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