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## Evaluating the Efficiency of Spacecraft Electric Thruster Operation

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### Abstract

The article analyses an ideal and a real model of thrust creation in ion plasma thrusters. The correction of the ideal ion plasma thruster model is proposed to be carried out with the help of efficiency factors. Factors affecting the efficiency of thrusters are analyzed.

Thrust to power ratio, which is one of the most important indicators of ion plasma thruster perfection, is evaluated as well.

**KEY WORDS:** Ion thrusters, plasma thrusters, losses, heat losses, efficiency, thrust, specific impulse

### 1. Introduction

The operating principle of plasmatron (electrothermal) spacecraft electric thrusters (SET) [1] is close to the operating principle of traditional chemical rocket engines, in which jet thrust is created by a gaseous (or gasified) working fluid jet flowing out of a quasi-closed volume; the working fluid expands in a discharge chamber (an analogue of the combustion chamber of traditional engines) under the effect of heat released during arc discharge. In such SETs, the mechanism of structural element heating is similar to the mechanism of heating the combustion chamber and nozzle of traditional rocket engines.

One of the most important problems is a necessity of building a model of processes occurring in SETs in order to correctly evaluate the thermal power being generated during SET functioning. Once such a maximally accurate evaluation has been provided, it becomes possible to reduce the energy consumption, mass, dimensions and cost of the cooling system, which is important for a spacecraft.

This article analyses the processes occurring in spacecraft electric thrusters (SET); it also provides calculations of losses and describes their influence upon the total thermal power generated during SET operation. At the same time, it considers thrusters, in which the process of jet thrust creation is related to the ionization and further acceleration of working fluid ions – ion plasma thrusters (IPT) [2-4].

### 2. An Ideal and a Real Model of Thrust Generation in Ion Plasma Thrusters

From the equation, which connects jet power  $P_{jet}$  and ion plasma thruster (IPT) thrust  $T$ , it follows that thrust augmentation can be achieved by increasing thruster jet power without increasing the consumption of working fluid:

$$P_{jet} = \frac{T^2}{2m_i n_i}, \quad (1)$$

where  $n_i$  is the number of ions;  $m_i$  - is the mass of working fluid ion.

The energy of electric field accelerating the ion in the IPT is equal to the product of ion charge  $q_i$  and the difference of the potentials of this field (the voltage between the electrodes, the difference of potentials between which is a source of this field  $V_b$  expressed in volts). According to the law of energy conservation, after conversion to ion kinetic energy, field energy will be equal to ion kinetic energy:

$$E = V_b q_i = \frac{1}{2} m_i V_{exi}^2. \quad (2)$$

From (2), it is possible to express ion flow velocity:

$$V_{exi} = \sqrt{\frac{2q_i V_b}{m}}. \quad (3)$$

Ion beam current  $I_b$  expressed in amperes is equal to the product of ion charge  $q_i$  and the number of ions  $n_i$ , which can be expressed through the ion mass flow:

$$I_b = q_i \cdot n_i = q_i \cdot \frac{G_i}{m_i}.$$

Whence the ion mass flow can be expressed through the ion beam current:

$$G_i = I_b \cdot \frac{m_i}{q_i} \quad (4)$$

From Eq. (1), which determines the thrust of the thruster, by substituting (3) and (4) and taking into consideration the assumption that ions in the IPT have a single degree of ionization, i.e. a single charge is equal to  $e$ , there appears the following expression for IPT thrust (in Newtons):

$$T = \sqrt{\frac{2 \cdot m}{e}} \cdot I_b \cdot \sqrt{V_b}. \quad (5)$$

Such a form of representation is convenient because the first cofactor of expression (5)  $\sqrt{\frac{2 \cdot m}{e}}$  is a constant for each type of working fluid; thus, thrust is determined by the beam current intensity and by the square root of accelerating voltage value.

The value of constant  $\sqrt{\frac{2 \cdot m}{e}}$  is: for hydrogen =  $1.44537 \cdot 10^{-4}$ ; for helium =  $2.88042 \cdot 10^{-4}$ ; for neon =  $6.467598 \cdot 10^{-4}$ ; for argon =  $9.09981 \cdot 10^{-4}$ ; for krypton =  $1.31796 \cdot 10^{-3}$ ; for xenon =  $1.64968 \cdot 10^{-3}$ ; for air =  $7.75058 \cdot 10^{-4}$ .

For example, the expression for the thrust of xenon thruster (in millinewtons) acquires the following form:

$$T = 1.65 \cdot I_b \cdot \sqrt{V_b}. \quad (6)$$

It is obvious that thrust is growing along with the increase of working fluid atomic weight; at the same time, the ratio of thrust constants for hydrogen and xenon pairs and for helium and xenon is 11.414 and 5.727 respectively.

Equation (5) describes the ideal model of thrust generation in the IPT – when the ion beam does not have divergence, i.e. all ions move along the trajectories that are strictly parallel to the axis passing through the thrust vector.

Besides, when forming the expression for the thrust of the thruster, it was assumed that all the ions had a single degree of ionization, though in reality the degree of ionization has probability distribution.

In a general case, the equation of ion motion in an electromagnetic field in conditions of low-density plasma takes the following form [5]:

$$m_i \cdot \frac{dV_i}{dt} = q \mathbf{x} \mathbf{E} - \frac{VP_i}{n} - \frac{qj}{\sigma} \cdot (\mathbf{v}_{exi} \mathbf{x} \mathbf{B}), \quad (7)$$

where  $m_i$  is the ion mass;  $\mathbf{v}_{exi}$  is the ion flow velocity;  $q$  is the ion charge;  $\mathbf{E}$  is the electric field strength;  $VP_i$  is the ion pressure gradient;  $n$  is the plasma concentration;  $j$  is the electric current density;  $\delta$  is the plasma conductivity;  $\mathbf{B}$  is the magnetic induction. Here and further on,  $\mathbf{x}$  is a sign of vector product, while vector variables are marked in bold.

Expression (5) does not take into account positive effects of other forces described in equation (7), for example, the increase of ion kinetic energy due to electronic heating, as well as negative effects related to the inhomogeneity of IPT plasma flow, for example, ion beam dispersion at collisions with neutral atoms contained in the thruster plasma flow.

Generally, expression (5) qualitatively describes the mechanism of thrust generation based on general model (7) and can be applied as an ideal model for calculating the error-free characteristics of a thruster.

In general terms, expression (5) for real thruster characteristics can be presented as:

$$T = \gamma \cdot \sqrt{\frac{2 \cdot m}{e}} \cdot I_b \cdot \sqrt{V_b}, \quad (8)$$

where  $\gamma$  is the coefficient which takes into consideration the above described effects reducing the thruster efficiency:

$$\gamma = \alpha \cdot \beta \cdot \delta, \quad (9)$$

where  $\alpha$  is the coefficient of beam dispersion taking into consideration the dispersion of ion beam;  $\beta$  is the ionization

coefficient taking into consideration the degree of ion beam ionization;  $\delta$  is the dissipation coefficient taking into consideration heat effects, dispersion on neutral atoms, oscillatory processes in plasma, etc.

At the same time  $\lim \alpha = \lim \beta = \lim \delta = \lim \gamma = 1$ . In the ideal case,  $\gamma = 1$ .

In some particular cases, it is just required to create models for the calculation of the above coefficients.

For the ion beam with uniform conic divergence after exiting the thruster, with constant current density and uniform electric field, correction coefficient  $\alpha$  is equal to the cosine of the half mean angle of beam divergence  $\theta$  or to the cosine of angle  $\theta$  between the generator and ion beam symmetry axis. In this case, taking into account the divergence of ion beam, Eq. (5) takes the following form:

$$T = \cos \theta \cdot \sqrt{\frac{2 \cdot m}{e}} \cdot I_b \cdot \sqrt{V_b} . \quad (10)$$

For example, if the angle of beam divergence is equal to 60 degrees,  $\theta = 30^\circ$  and  $\cos \theta = 0.866$ , i.e. the loss of thruster thrust due to beam divergence is 13.4%.

If the source of plasma does not provide a uniform plasma flow, i.e. the above listed conditions related to current density, field and beam geometry are violated, the correction coefficients have to be integrated for the given surfaces with account of their curvatures.

In this case, for cylindrical thrusters:

$$\alpha = \frac{1}{I} \int_0^r 2\pi \cdot J(r) \cdot \cos \theta(r) dr, \quad (11)$$

where  $J(r)$  is the function of ion current density depending on radius  $r$ .

When  $J(r) = \text{const}$ , expression (11) transforms to (10).

In practice, the distribution of ion current can be measured during the experimental development and testing of thrusters with the help of Langmuir probes or other similar devices.

Ionization coefficient  $\beta$  correction takes into account the presence of multicharged ions in plasma.

If thruster plasma simultaneously contains singly charged, doubly charged and triply charged ions, the total beam current is equal to the sum of currents that correspond to the flows of singly charged, doubly charged and triply charged ions:

$$I_b = I^+ + I^{++} + I^{+++} , \quad (12)$$

where  $I^+$ ,  $I^{++}$  and  $I^{+++}$  are the currents of the flows of singly charged, doubly charged and triply charged ions respectively.

In this case, the total thrust generated by the flows of singly charged, doubly charged and triply charged ions will be:

$$T = \sqrt{\frac{V_b m}{e}} \cdot I^+ \left( 1 + \frac{1}{\sqrt{2}} \cdot I^{++} / I^+ + \frac{1}{\sqrt{3}} \cdot I^{+++} / I^+ \right) . \quad (13)$$

Then ionization coefficient  $\beta$  can be expressed as follows:

$$\beta = \left( 1 + \frac{1}{\sqrt{2}} \cdot I^{++} / I^+ + \frac{1}{\sqrt{3}} \cdot I^{+++} / I^+ \right) . \quad (14)$$

In practice, the ion current created by singly charged, doubly charged and triply charged ions can be measured with the help of a magnetic sector charge analyzer during the experimental development of the IPT.

Dissipation coefficient  $\delta$  is to a considerable extent determined by two groups of factors:

– by the ratio of the number of ionized atoms to the total number of working fluid atoms, or by the ionization coefficient;

– by the inhomogeneity of magnetic and electric field in the IPT.

The development of a reliable mathematical model for determining dissipation coefficient  $\delta$  poses a difficult challenge because the reliable model has to describe various types of plasma flow inhomogeneities, the inhomogeneities of magnetic and electric fields and their fluctuations for different Volt-Ampere and thermal modes of the thruster, etc.

Such models are developed for series-produced IPTs on the basis of empirical data acquired as a result of trial operation and experimental development by measuring the spatial distribution of the electric, magnetic and thermal field of an operating thruster. In the long run, this sort of modelling reduces to determining heat losses, so this type of research is usually carried out while developing thruster thermal models based on experimental data, as for instance in

work [5].

Dissipation coefficient  $\delta$  can be expressed as:

$$\delta = k_1 \cdot N_i + k_2, \quad (15)$$

where  $k_1$  is the coefficient taking into consideration the interaction of neutral atoms and ions;  $N_i$  is the degree of ionization;  $k_2$  is the inhomogeneity coefficient.

Another method of determining dissipation coefficient  $\delta$  is obtaining its value when determining the efficiency factor and utilization factors [6].

### 3. Conclusion

The approach proposed in the article allows to develop the mathematical models of IPT operation, determine IPT losses and efficiency, it also allows to make the equation of IPT heat balance more precise and achieve sufficient accuracy when performing calculations on IPT thermal loads and cooling system.

This approach will allow to avoid overheating and destruction of high-power spacecraft electric thrusters, increase their thrust and specific impulse as well as to optimize the cooling system in the course of development and testing.

### References

1. **Brown, D.** 2009. "Investigation of low discharge voltage hall thruster characteristics and evaluation of loss mechanisms". A dissertation submitted in fulfillment of the requirements for the degree of Doctor of Philosophy (Aerospace Engineering) in The University of Michigan).
2. **Goebel, D.; Katz, I.** 2008. «Fundamentals of Electric Propulsion: Ion and Hall Thrusters», Pasadena: Jet Propulsion Laboratory, California Institute of Technology.  
([http://descanso.jpl.nasa.gov/SciTechBook/series1/Goebel\\_cmprsd\\_opt.pdf](http://descanso.jpl.nasa.gov/SciTechBook/series1/Goebel_cmprsd_opt.pdf))
3. **Jonathan, L. Van Noord.** 2010. "NEXT Ion Thruster Thermal Model". Glenn Research Center, Cleveland, Ohio, NASA/TM—2010-216919, <https://ntrs.nasa.gov/archive/nasa/casi.ntrs.nasa.gov/20110000534.pdf>
4. **Morozov, A.; Savelyev, V.** 2000. «Fundamentals of Stationary Plasma Thruster Theory». ISBN: 978-0-306-11064-1. Springer US (Boston, MA).
5. Encyclopedic Series "Encyclopedia of Low Temperature Plasm" Edited by V. E. Fortov, Introduction Volume, Book IV, XI.4 Ion and Plasma Rocket Thrusters., Moscow: Nauka publishers, 2000.
6. "Electric Propulsion Innovation & Competitiveness" (EPIC) project ([http://epic-src.eu/?page\\_id=63](http://epic-src.eu/?page_id=63))