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Теоретические исследования и практическое использование вибрационных процессов в технике и технологиях являются актуальной сферой деятельности и научных интересов многих ученых в России и за рубежом.

Результаты этих работ нашли свое отражение в программе X Международной научно-технической конференции «Вибрация-2012. Управляемые вибрационные технологии и машины».

Проведение конференции в Юго-Западном государственном университете стало уже хорошей традицией и закономерностью, подтверждающей успешное развитие научной школы по вибрационной механике, заложенной заслуженным деятелем науки и техники России, профессором П.М. Алабужевым. В сборнике, публикуемом по итогам ее работы, представлены результаты исследований ведущих ученых России, Германии, Франции, Непала, Латвии, Украины, Белоруссии.

Тематика представленных на конференции научных работ весьма широка и многогранна: история развития механики, динамика конструкций и машин, моделирование динамических процессов, виброзащита, волновые процессы и случайная вибрация, мехатроника, робототехника и биомеханика. Особое внимание уделено системам, имеющим автоматическое управление и регулирование.

Сборник будет полезен научным работникам, инженерно-техническим специалистам, аспирантам и студентам, занимающимся проблемами исследований в области динамики машин и разработкой современной вибрационной техники и вибрационных технологий.

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SOME PROPERTIES OF NONLINEAR LATERAL OSCILLATIONS OF FLEXIBLE ELEMENTS UNDER PARAMETRIC EXCITATION

Lateral oscillations of flexible elements (belts, cables, guy ropes, strings, etc.) in machines and devices under parametric excitation are studied. Mathematically the problem is presented as a partial differential equation describing parametric oscillations of flexible element with due account of its geometrical, static and physical nonlinearities. Two different methods of analysis have been used: mathematical simulation on analogue-digital computer system and numerical solution with computer programme MATLAB. It is shown that under some conditions resonant parametric oscillations of flexible element occur within wide continuous frequency zones. Besides, abrupt changes of amplitudes and modes of oscillations become possible within these zones. In such conditions system's tuning away from dangerous resonant regimes remains problematic. The possible ways for practical application of these nonlinear effects are considered.

Flexible elements (belts, cables, guy ropes, taut threads, strings, etc.) are widely used in mechanical engineering for various practical purposes [1, 2]. In some cases lateral parametric vibrations of flexible elements, which can occur during the operation of machine, may be extremely detrimental (belt and chain transmissions, threads in weaving machines, etc.). But in some other cases special excitation of lateral oscillations of flexible element is needed to ensure effective operation of vibration device (for example, flexible belt of vibration mixer, vibration setup for seismic ground probing, elastic tail of robotic fish, etc.). Therefore it is very important on preliminary stage of designing to analyze different resonant regimes which can occur in the system under periodic pulsation of axial tension force.

Most of known works on non-linear oscillations of flexible elements are concerned with the analysis of free vibrations (e.g., [3-5]). But cases of parametric excitation are usually considered in application only to the first lower mode of string's lateral oscillations [6-8]. This paper seeks to study high-frequency lateral modes of parametric oscillations. On the base of this study new practical applications are possible.

Dynamic model and methods of analysis. Lateral oscillations of flexible element under parametric excitation are considered (Fig. 1). Parametric excitation is caused by periodic variation in time of axial tension force of the flexible element. In forming of differential equation of oscillations some assumptions are made. It is supposed, that stiffness in bending of flexible element is negligible in comparison with its stiffness in tension, but weight of flexible element is ignorable in comparison with axial prestressing force T_0. Besides, it is considered that oscillations are performed in z-y plane, which runs along the centre line of a non-deformed flexible element.

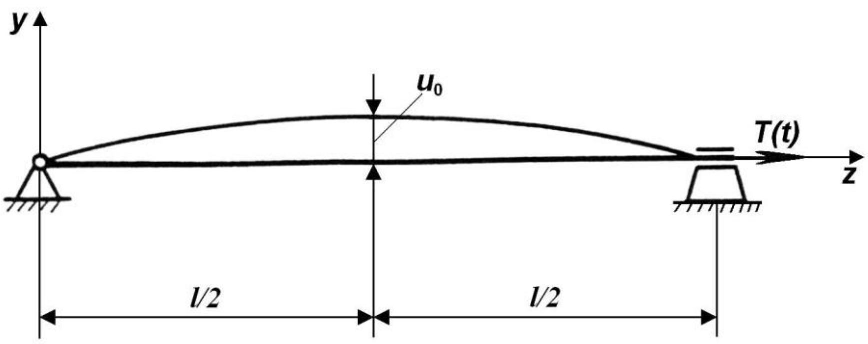

Fig. 1. Model considered in dynamic analysis

Taking the direction of co-ordinate axis z along this centre line, the differential equation for lateral vibrations of flexible element can be stated as follows [2, 5]:

$$T_0\left(1+\mu\sin\Omega t\right)\left[1+f(\varepsilon)\right]\left(\frac{\partial^2 y}{\partial z^2}+b_1\frac{\partial^3 y}{\partial z^2\partial t}\right)-b_2\frac{\partial y}{\partial t}-\rho\left[1+\frac{1}{2}\left(\frac{\partial y}{\partial z}\right)^2\right]\frac{\partial^2 y}{\partial t^2}=0\,, \qquad 1)$$

where T_0 is the prestressing force; μ and Ω are the non-dimensional amplitude and the frequency of parametric excitation; accordingly b_1 and b_2 are the coefficients of internal and external frictions; *y* is the lateral displacement of the flexible element.

The functional *f*(ε) in equation (1) takes into account additional tension caused by elastic deformation of flexible element during its oscillations (physical non-linearity). The elongation ε of flexible element can be determined by formula $\varepsilon = \frac{1}{2l}\int_0^l \left(\frac{\partial y}{\partial z}\right)^2 dz$ [5], where *l* is the length of flexible element. The relationship between axial stress σ in flexible element and its elongation ε can be approximately described by the expression $\sigma = E\varepsilon - \beta\varepsilon^3$, where *E* is the elasticity modulus of material; β is the coefficient of non-linearity.

In this case the functional *f*(ε) can be expressed in the following form

$$f(\varepsilon) = \frac{EA}{2T_0 l}\int_0^l \left(\frac{\partial y}{\partial z}\right)^2 dz - \frac{\beta A}{8T_0 l^3}\left[\int_0^l \left(\frac{\partial y}{\partial z}\right)^2 dz\right]^3, \qquad 2)$$

where *A* is the cross-section area of flexible element.

Therefore an increment in tension is caused by integral elongation of flexible element and is independent of co-ordinate *z*. Non-linear term $\left[1 + \frac{1}{2}\left(\frac{\partial y}{\partial z}\right)^2\right]$ of equation (1) takes into account geometrical non-linearity of flexible element [5]. In the case studied here the end boundary conditions are as follows:

$$y(z=0,t)=0\ ; \qquad y(z=l,t)=0\,. \qquad 3)$$

Equation (1), subject to the conditions (2) – (3), was solved in MATLAB environment using *pdepe* solver [9]. For this purpose initial differential equation (1) was transformed into equivalent set of first-order partial differential equations, and special-purpose MATLAB functions describing nonlinear characteristics have been created. Dynamics of the system also has been simulated on the specialized analogue-digital computer system developed in Riga Technical University [10] and some of these theoretical results have been verified by experiments with a uniform rubber cord. The methods of mathematical simulation and the operational principle of the computer system are described in more detail in [2, 11].

Analysis of parametric oscillations of flexible element. As known [12, 13], parametric resonance of flexible element occur under periodic pulsation of tensile force *T* with frequency Ω which fall in the vicinity of critical frequencies

$$\Omega = \frac{2\omega_S}{e} \quad \text{or} \quad \Omega = \frac{|\omega_S \pm \omega_k|}{e}. \qquad 4)$$

where ω_S and ω_k are the natural frequencies with ordinal numbers S and k of lateral oscillations of flexible element; $e = 1, 2, 3 \ldots$ is the order of parametric resonance. The first formula describes the condition of a simple parametric resonance, but the second formula – condition of a combined parametric resonance.

In accordance with the conditions (4), there are possible some critical frequencies Ω which theoretically allow excitation both simple and combination parametric resonances. For example, such situation occurs under the conditions $\Omega = 2\omega_2 = \omega_1 + \omega_3$, $\Omega = 2\omega_6 = \omega_5 + \omega_7$, etc. In this case, as it is shown by the mathematical simulation [14], simple parametric oscillations dominate over the combination ones. Besides, under some conditions ($\mu > 0.15$ and $\eta > 11$) main zones of simple parametric regimes $2\omega_s$ are merging, and as the result a continuous resonant zone occurs in wide frequency range.

As an example, Fig. 2 shows a part of the diagram of parametric instability corresponding to the frequency range of excitation of regimes $2\omega_6$, $2\omega_7$ and $2\omega_8$. The diagram is constructed on the plane of parameters μ and $\eta = \Omega/\omega_1$, assuming $b_1\omega_1 = 0{,}003$. Above on the same figure AFC of lateral parametric oscillations of flexible element is constructed (for the case $\mu = 0{,}2$, $T_0/EA = 2\cdot10^{-4}$ and $b_1\omega_1 = 0{,}003$). Dimensionless peak value of displacement u_0/l in antinodal point of corresponding resonant mode is projected as amplitude on this AFC.

In accordance with the diagram presented, an ambiguity of exciting parametric regimes exists inside frequency ranges *bc* and *ef*. Dependent on realized initial conditions, regimes of orders $2\omega_6$ (6^{th} mode) or $2\omega_7$ (7^{th} mode) can be excited inside frequency range *bc*. But inside frequency range *ef* it is possible to realize oscillations of flexible element by 7^{th} or 8^{th} mode. Besides, due to nonlinear properties of flexible element pulling of oscillations beyond the borders of instability zone occurs (frequency intervals *ce* and *fg*). Abrupt changes of amplitudes and modes of oscillations are observed in bifurcation points (e.g. bifurcation jumps $b''-b'$, $d'-d''$, $e'''-e''$, $g''-g'''$).

Typical parametric regimes of oscillations realized within instability zones $2\omega_6$ and $2\omega_7$ are presented in Fig. 3 and Fig. 4, obtained by numerical investigations with computer programme MATLAB. As it is seen, during transient process shapes of vibration modes gradually approach to the shapes peculiar to 6^{th} and 7^{th} natural modes.

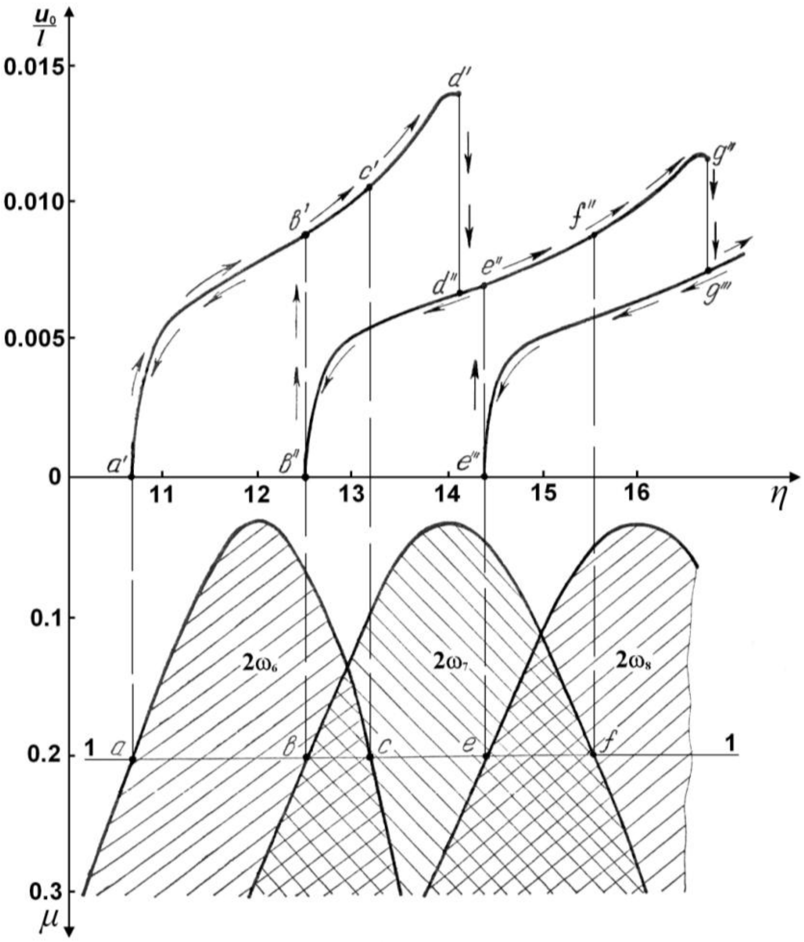

Fig. 2. Overlap of parametric instability zones under the high-frequency excitation

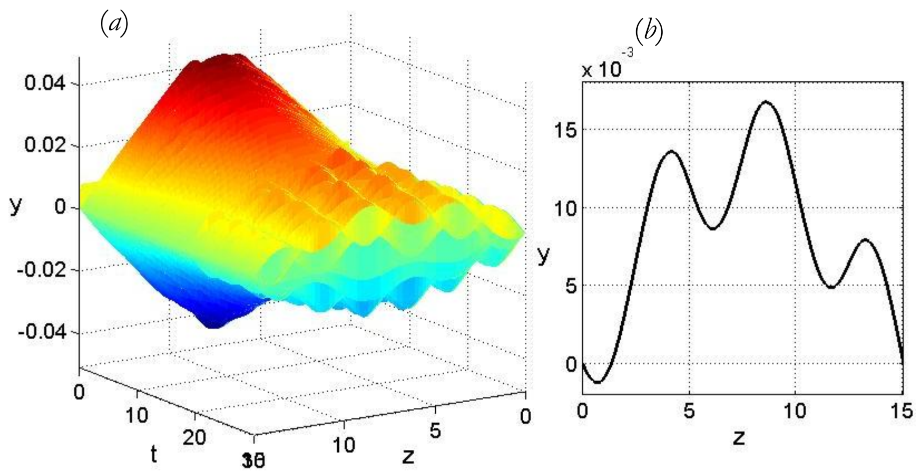

Fig. 3. Typical parametric regimes of oscillations within instability zone $2\omega_6$:

(*a*) transition to a steady-state oscillations from non-zero initial conditions;

(*b*) vibration mode at instant of time t = 30 s

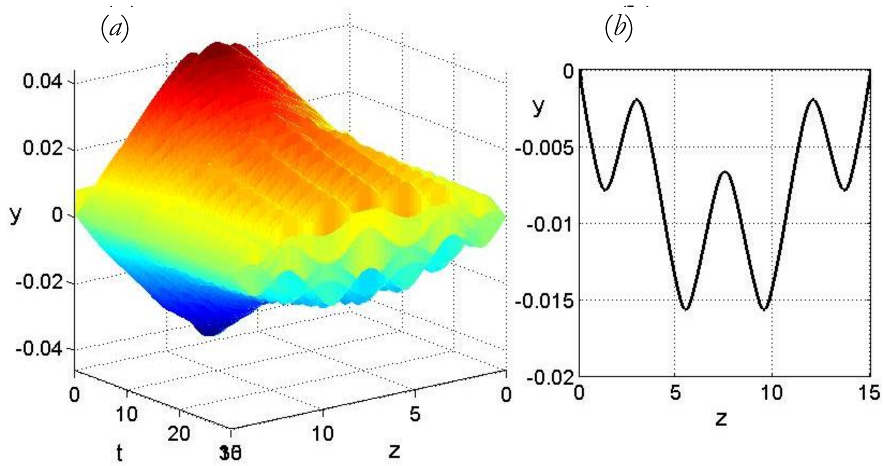

Fig. 4. Typical parametric regimes of oscillations within instability zone $2\omega_7$:
(*a*) transition to a steady-state oscillations from non-zero initial conditions;
(*b*) vibration mode at instant of time t = 30 s

Some distortions of vibration modes (see Fig. 3 and Fig. 4) from the standard harmonic shapes can be caused by the action of two different factors. First, the influences of geometric and static nonlinearities of flexible element have to be taken into consideration. Second, the interaction of different mechanisms of excitation of parametric regimes may be the reason for distortion of vibration modes. For example, due to the condition (4) at the excitation frequency $\Omega = 2\omega_6$ there is a theoretical possibility for additional exciting of combination parametric regimes of orders $\omega_5 + \omega_7$, $\omega_4 + \omega_8$, $\omega_3 + \omega_9$, $\omega_1 + \omega_{11}$ and others. Interaction of simple parametric and combination regimes can result in specific distortions of vibration modes and frequency spectrum. These effects are needed in further more detailed analysis.

Results of the theoretical study have been verified by experiments with a uniform rubber cord having length l = 0.66 m and linear density ρ = 0.0025 kg/m (in unloaded condition). One end of the flexible element was fixed, but the other one was connected with the slide-block of a crank gear (through the system of guiding rolls). During the reciprocation of slide-block a tension force of flexible element was periodically changed, and thanks to these lateral parametric oscillations of cord were excited. Existence of wide continuous resonant frequency ranges and occurrence of vibration regimes accompanied with abrupt changes of amplitudes and modes of oscillations were confirmed.

Existence of resonant parametric oscillations of flexible element within wide continuous frequency range (Fig. 2) makes some difficulties in vibration protection. In such cases system's tuning away from dangerous resonant regimes remains problematic. But from the other hand, special excitation of these regimes can be useful in vibration devices intended for executing of some technological processes with the aid of vibrating flexible element (flexible belt of vibration mixer, elastic tail of robotic fish, emptying of loading hoppers with the aid of vibrating belt, etc.).

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