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**ВИБРАЦИОННЫЕ ТЕХНОЛОГИИ, МЕХАТРОНИКА  
И УПРАВЛЯЕМЫЕ МАШИНЫ**

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Теоретические исследования и практическое использование вибрационных процессов в технике и технологиях являются актуальной сферой деятельности и научных интересов многих ученых в России и за рубежом.

Результаты этих работ нашли свое отражение в программе XI Международной научно-технической конференции «Вибрация-2014. Вибрационные технологии, мехатроника и управляемые машины».

Проведение конференции в Юго-Западном государственном университете стало уже хорошей традицией и закономерностью, подтверждающей успешное развитие научной школы по вибрационной механике, заложенной заслуженным деятелем науки и техники России, профессором П.М. Алабужевым. В сборнике, публикуемом по итогам ее работы, представлены результаты исследований ведущих ученых России, Германии, Франции, Непала, Латвии, Украины, Белоруссии.

Тематика представленных на конференции научных работ весьма широка и многогранна: история развития механики, динамика конструкций и машин, моделирование динамических процессов, виброзащита, волновые процессы и случайная вибрация, мехатроника, робототехника и биомеханика. Особое внимание уделено системам, имеющим автоматическое управление и регулирование.

Сборник будет полезен научным работникам, инженерно-техническим специалистам, аспирантам и студентам, занимающимся проблемами исследований в области динамики машин и разработкой современной вибрационной техники и вибрационных технологий.

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**J. Viba, D. Vitols, S. Cifanskis, V. Beresnevich, S. Noskovs***Riga Technical University (Латвия)***INVESTIGATION OF OBJECT VIBRATION INTERACTION WITH FLUID IN SMALL VELOCITY REGION**

*Annotation. In the robots techniques investigators all time have interaction with continue media like air and water flow. Very large achievements in mechanics of the field of aero and hydro dynamics achieved. In the same time there are many technical tasks to be solved. One of the tasks group is synthesis propulsion systems using vibrations of objects elements inside flow. Here can be taken into account motion direction exchanges into small velocity region when body elements start vibration motion. In this report technical theory for modelling of object motion in small velocity region is investigated. Two degree of freedom propulsion system with adaptive control is investigated.*

**1. Introduction**

Modern engineering often has to solve problems of interaction between fluids and solid bodies. Word “interaction” basically means forces and moments acting on body in fluid flow. Also, important is question about pressure distribution on body surface. It’s hard to solve such problems in analytic manners, so the only way to get correct and practically useful results is to do experimental research or to simulate flow using computational fluid dynamics (CFD) software. However, here we will describe simplified approach to calculate aerodynamic forces analytically, using theorem of momentum exchange [1 – 4].

It’s convenient to present resulted force with two components: Drag force (acting along flow direction) and Lift force (acting in perpendicular direction). The following equations are used to calculate these forces [5, 6]:

$$F_d = C_d \cdot S \cdot \frac{\rho \cdot V^2}{2} ; \quad F_l = C_l \cdot S \cdot \frac{\rho \cdot V^2}{2} , \quad (1)$$

where  $\rho$  – density,  $V$  – flow velocity,  $S$  – specific area,  $C_d$  and  $C_l$  – drag and lift coefficients. These coefficients depend on bodies’ geometry, orientation relative to flow, and non-dimensional Reynolds number:

$$Re = \frac{d \cdot V}{\nu} , \quad (2)$$

where  $d$  – specific size,  $\nu$  – kinetic viscosity. As example, typical drag coefficient dependence of  $Re$  for a sphere is shown on Figure 1 [7].

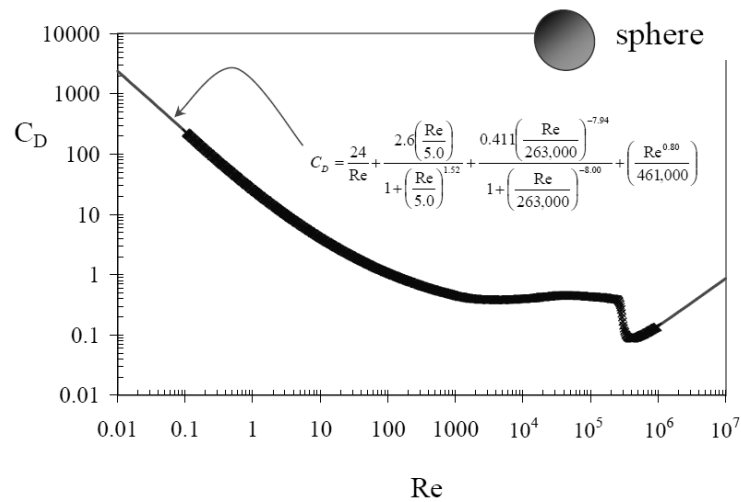


Fig. 1. Drag coefficient dependence on Reynolds number for a sphere [7]

Drag and lift coefficients for different shapes of bodies can be determined experimentally using wind tunnels.

**2. Simplified theoretical model**

**2.1. Flat plate**

First attempts to determine force acting on plate perpendicular to flow were made long time ago. Consider fluid particles fully lose kinetic energy after hitting obstacle. Kinetic energy of fluid volume is (Fig.2):

$$\rho \cdot L \cdot S \cdot \frac{V^2}{2} . \tag{3}$$

Plate stops this volume, thus revealing useful work:

$$A = L \cdot F . \tag{4}$$

From equations (3) and (4) yields:

$$F = S \cdot \frac{\rho \cdot V^2}{2} . \tag{5}$$

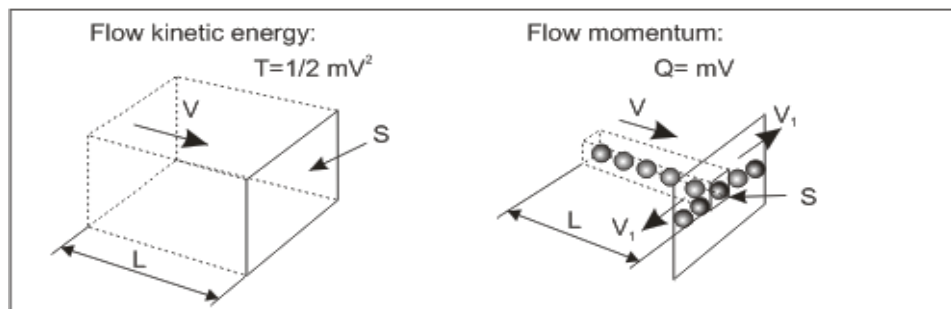


Fig. 2. Two approaches to determine force acting on plane

We consider it's not being truly correct to use the theorem of kinetic energy exchange, because actually, after hitting surface, particles don't stop, but go in all directions, thus keeping some energy. Kinetic energy is scalar, and it can't be projected on flow direction. Instead, we recommend using theorem of momentum exchange:

$$m \cdot \bar{v}_2 - m \cdot \bar{v}_1 = \int_0^{\tau} \bar{F} \cdot dt \quad , \quad (6)$$

where  $m = \rho \cdot L \cdot S$  – elementary volume mass,  $\tau = L/V$  – time interval in which mass hits the plate. Consider here  $v_2 = 0$  and  $v_1 = V$ , the following can be yielded:

$$F = S \cdot \rho \cdot V^2 \quad . \quad (7)$$

The only difference between (5) and (7) is coefficient “1/2”.

However, using (5) or (7) in practice don't give corresponding results. Even for a simple object like plate (5) gives 20% error. That's because drag and lift coefficients are used. They show how much real force differs from calculated with (5). Some examples of drag coefficients for several basic shapes are shown below.

So, it's not important to use the “1/2” coefficient or not, but one must remember, that all experimentally obtained aerodynamic coefficients were calculated using (1), that includes this coefficient. Likely, formula (1) is used in practice, because it accents the force dependency on flow kinetic energy proportional  $V^2$ .

Formula (7), however, gives satisfying results when calculating force acting on immobile wall.

## 2.2. Solid body

Consider more general case, where momentum exchange theorem is used (Fig. 3.). For elementary area  $dx \cdot dy$  the following can be written:

$$dm \cdot \bar{V}_1 - dm \cdot \bar{V}_0 = d\bar{F} \cdot dt \quad . \quad (8)$$

Here  $dm = \rho \cdot V_0 \cdot dt \cdot dx \cdot dy \cdot \cos \gamma$ . If velocity after impact is proportionally linear to the velocity before, interaction follows:

$$V_1 = k1 \cdot V_0 \cdot \sin \gamma \quad . \quad (9)$$

Here, coefficient  $k1$  includes boundary layer interaction forces (and viscous damping forces too).

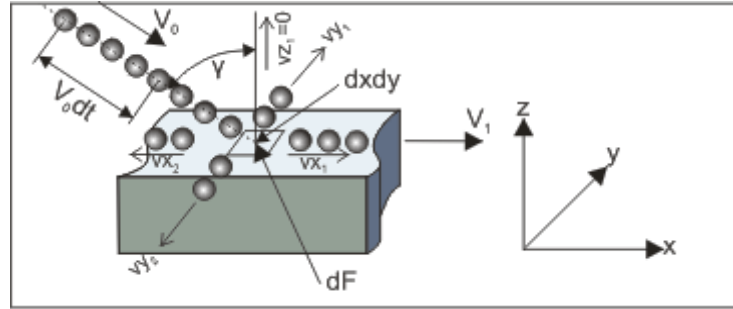


Fig. 3. Finding force acting on solid body in general case

Projecting the force to axis we get:

$$x: dF_x = -\rho \cdot V_0^2 \cdot dx \cdot dy \cdot \cos\gamma \cdot \sin\gamma \cdot (1 - k_1); \quad (10)$$

$$y: dF_y = 0; \quad (11)$$

$$z: dF_z = \rho \cdot V_0^2 \cdot \cos^2\gamma \cdot dx \cdot dy. \quad (12)$$

Equations (9) – (11) can be used for global fluid environment resistance  $F$  calculation. In this case integrals along surface  $f = f(x, y, z)$  must be found, take into account, that angle  $\gamma$  along streamlines may be function of  $x, y$ . Additionally, velocity  $V_0$  may vary too.

### 2.3. Rotating plate

Consider constant width plate fixed to some housing with a joint (Fig.4). Plate is rotating around the joint with constant radial velocity  $\omega$ . Goal is to calculate reactions and moment in the joint. We can write equation (8) for elementary mass  $dm$ . Then  $V_1 = \omega r$ ,  $V_0 = 0$ ,  $dm = \rho b \omega r dr dt$ . The elementary impulse of force  $dF$  acting on area  $b \cdot dr$  is as follows:

$$dF \cdot dt = \rho \cdot b \cdot \omega^2 \cdot r^2 \cdot dr \cdot dt. \quad (13)$$

Total force  $F$  and rotation moment  $Mo(dF)$  of distributed forces can be found by integrating along the length of the plate:

$$F = -\int_0^L \rho \cdot b \cdot \omega^2 \cdot r^2 \cdot dr \cdot \text{sign}(\omega) = -\frac{\rho \cdot b \cdot \omega^2}{3} L^3 \cdot \text{sign}(\omega);$$

$$Mo(dF) = -\int_0^L \rho \cdot b \cdot \omega^2 \cdot r^2 \cdot r \cdot dr \cdot \text{sign}(\omega) = -\frac{\rho \cdot b \cdot \omega^2}{4} L^4 \cdot \text{sign}(\omega). \quad (14)$$

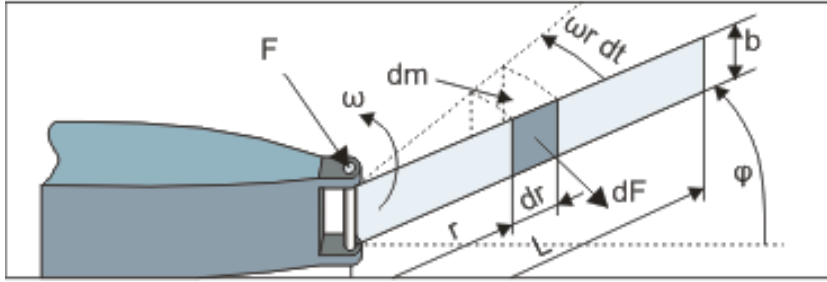


Fig. 4. Plate rotating in fluid environment with angular velocity  $\omega$

This force is perpendicular to plate, so reaction components can be found:

$$F_x = F \cdot \sin \varphi \cdot \text{sign}(\omega) , \quad F_y = -F \cdot \cos \varphi \cdot \text{sign}(\omega) . \quad (15)$$

Here, direction of forces is considered with velocity signature functions  $\text{sign}(\omega)$  .

### 3. Aerodynamic forces at small $Re$ number

Additionally, we will show a convenient expression, that approximates drag force dependency on flow velocity at small  $Re$  values. As you can see on Fig.1, drag coefficient (for a sphere) is constant in large  $Re$  interval. However, at small  $Re$  values  $C_d$  are much higher. We found, that force dependency on the velocity in this interval with direction exchange can be described as following power function expression:

$$F1 = \frac{C1}{V^a} V^2 = C1 \cdot V^{(2-a)} .$$

Here  $C1$  and  $a$  – constants. So, at small  $Re$  values formula (16) can be used instead of (1). Additionally for vibration motion with velocity directions exchanges can be used function:

$$F1 = -C1 \cdot |V^{(2-a)}| \cdot \text{sign}(V) . \quad (16)$$

Inverse sign graphics of function (16) (for special value of constant  $a$ ) are shown in Fig. 5. All graphics cross three points with velocity argument  $V = 0, V = \pm 1$ .

There from analyzing formulas (7), (16) and graphics Fig. 5. vibration interaction can be expressed as function:

$$F1 = -C1 \cdot \left| V^{2-a(0,5-0,5 \cdot \text{sign}(|V|-1))} \right| \cdot \text{sign}(V) . \quad (17)$$

Inverse sign graphics of function (17) are shown in Fig. 6.

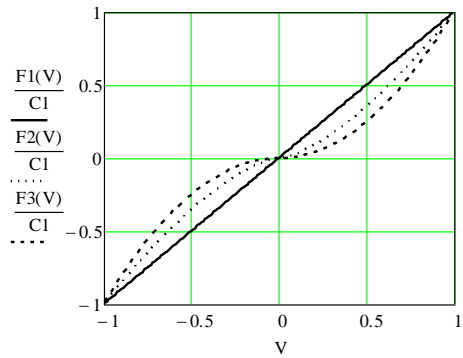


Fig. 5. Aerodynamic vibration force at small Re number: F1 for  $a = 1$ ; F2 for  $a = 0,5$ ; F3 for  $a = 0$ .

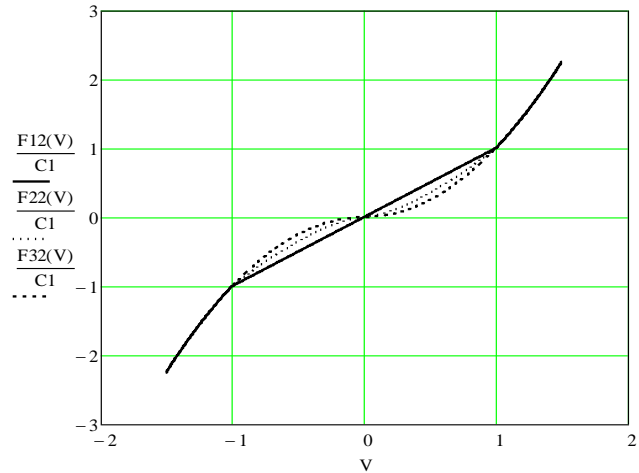


Fig. 6. Vibration interaction force at all velocity region: F12 for  $a = 1$ ; F22 for  $a = 0,5$ ; F33 for  $a = 0.5$

#### 4. Modelling of two degree of freedom system

Model of pure translation motion of two degree of freedom system is shown in Fig. 7. The equations of motion are (18):

$$\begin{aligned}
 m_1 \ddot{x}_1 &= -C_1(\dot{x}_1)^2 \text{sign}(\dot{x}_1) - c_{12}(x_1 - x_2) - b_{12}(\dot{x}_1 - \dot{x}_2) + P_0 \sin(\omega t); \\
 m_2 \ddot{x}_2 &= -C_0(1 - k \cdot \text{sign} \dot{x}_2) \dot{x}_2^{2-a(0,5-0,5\text{sign}(|\dot{x}_2|-1))} \left| \text{sign}(\dot{x}_2) + c_{12}(x_1 - x_2) + b_{12}(\dot{x}_1 - \dot{x}_2) - P_0 \sin(\omega t) \right|, \quad (18)
 \end{aligned}$$

were  $m_1, m_2$  – masses;  $x_1, x_2, \dot{x}_1, \dot{x}_2, \ddot{x}_1, \ddot{x}_2$  - displacements, velocities and accelerations of masses;  $C_1, C_0, c_{12}, b_{12}, P_0, \omega, k, a$  – constants,  $t$  – time.

As control action here adaptive exchange of cross section of tail area is used:  $-C_0(1 - k \cdot \text{sign} \dot{x}_2)$  [8].

Examples of motion in phase plane for masses  $m_1$  and  $m_2$  are shown in Fig. 8.

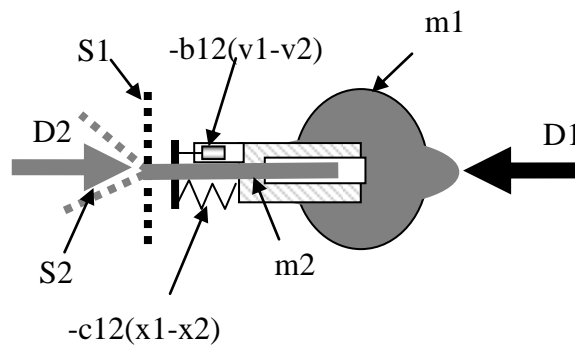


Fig. 7. Two degree of freedom propulsion system



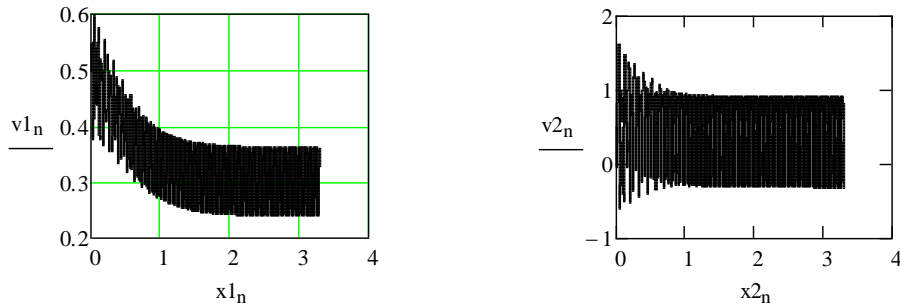


Fig. 8. Motion in phase plane of masses  $m_1$  and  $m_2$ . The average speed is about 30 cm/s

## 6. Conclusions

Simple theoretical approaches like described above are not usually used, because they don't consider many conditions of a real fluid motion. To obtain correct results, this simple theory must be supplied with additional expressions that consider other important parameters. Theorem of momentum exchange only describes interaction between fluid particles and obstacles and does not consider internal interaction between fluid particles, boundary layer on body surface, viscosity, turbulence and other things.

## REFERENCES

1. Targ S.M. Short course of theoretical mechanics. Moscow: High school, 1986. 416 p.
2. Loyciansky L.G. Fluid mechanics. Moscow: Drofa, 2003. 840 p.
3. Sedov L.I. Planar problems of hydro- and aerodynamics. Moscow: Nauka, 1966. 448 p.
4. Panovko Y.G. Introduction to mechanical oscillations theory. S.-Petersburg: Nauka, 1989. 252 p.
5. Clancy L.J. Aerodynamics. Publisher by Pitman. New York; London, 1975. 610 p.
6. Morrison F.A. An Introduction to Fluid Mechanics / Cambridge University Press. New York, 2013. 625 p.
7. URL: <http://www.chem.mtu.edu/~fmorriso/DataCorrelationForSphereDrag2013>.
8. Viba J. Optimisation and synthesis of vibro impact systems. Riga: Zinatne, 1988. 253 p.